## Blackwater Community School Curriculum Map 2015-2016

## Fourth Grade Quarter 2

## Module 3: Multi-Digit Multiplication and Division - Part 2, Topics E-H <br> Approximately 28 days - Begin around October $13^{\text {th }}$

In this 43-day module, students use place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. As a key area of focus for Grade 4, this unit moves slowly but comprehensively to develop students' ability to reason about the methods and models chosen to solve problems with multi-digit factors and dividends.

| Major Clusters: |  |  | 4.OA.A - Use the four operations with whole numbers to solve problems. <br> 4.NBT.B - Use place value understanding and properties of operations to perform multi-digit arithmetic. |  |  |
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| Supporting Clusters: |  |  | 4.OA.B - Game familiarity with factors and multiples. |  |  |
| Vocabulary |  |  | Associative property, composite number, distributive property, divisor, partial product , prime number, remainder, solve, principle of counting, organize, random, possibilities, similarities, differences, chart/arrays, systematic lists, tree diagram, outcomes, systematic list |  |  |
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| 4.0A | A | 3 | Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Focus on addition and subtraction.)(Q1, Q3) <br> 4.MP.1. Make sense of problems and persevere in solving them. <br> 4.MP.2. Reason abstractly and | Students need many opportunities solving multistep story problems using all four operations. <br> An interactive whiteboard, document camera, drawings, words, numbers, and/or objects may be used to help solve story problems. <br> Example: <br> - Chris bought clothes for school. She bought 3 shirts for $\$ 12$ each and a skirt for $\$ 15$. How much money did Chris spend on her new school clothes? $3 \times \$ 12+\$ 15=a$ <br> In division problems, the remainder is the whole number left over when as large a multiple of the divisor as possible has been subtracted. <br> Example: <br> - Kim is making candy bags. There will be 5 pieces of candy in each bag. She had 53 pieces of candy. She ate 14 pieces of candy. How many candy bags can Kim make now? | Engage NY <br> M3 Lessons 14-21, 26- <br> 38 <br> Also addressed in Module 7 <br> enVision <br> Topic 1,5,10 |


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|  |  |  | quantitatively. <br> 4.MP.4. Model with mathematics. <br> 4.MP.5. Use appropriate tools strategically. <br> 4.MP.6. Attend to precision. <br> 4.MP.7. Look for and make use of structure. | (7 bags with 4 leftover) <br> - Kim has 28 cookies. She wants to share them equally between herself and 3 friends. How many cookies will each person get? (7 cookies each) $28 \div 4=a$ <br> - There are 29 students in one class and 28 students in another class going on a field trip. Each car can hold 5 students. How many cars are needed to get all the students to the field trip? ( 12 cars, one possible explanation is 11 cars holding 5 students and the $12^{\text {th }}$ holding the remaining 2 students) $29+28=11 \times 5+$ 2 <br> Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to: <br> - front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts), <br> - clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), <br> - rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), <br> - using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000), using benchmark numbers that are easy to compute (student's select close whole numbers for fractions or decimals to determine an estimate). |  |
| 4.0A | AZ | 3.1 | Solve a variety of problems based on the multiplication principle of counting. <br> a. Represent a variety of counting problems using arrays, charts, and | As students solve counting problems, they should begin to organize their initial random enumeration of possibilities into a systematic way of counting and organizing the possibilities in a chart (array), systematic list, or tree diagram. They note the similarities and differences among the representations and connect them to the multiplication principle of | Engage NY Not covered enVision Not covered |



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|  |  |  |  | Sample conclusions: <br> o There are 18 different dinner choices that include a meal, a drink, and a cupcake. <br> o Nine dinner choices are possible for the guest that wants spaghetti for her meal. <br> o A guest cannot choose a meal, no drink, and two cupcakes. <br> - Use multiple representations to show the number of meals possible if each meal consists of one main dish and one drink. The menu is shown below. Analyze the various representations and describe how the representations illustrate the multiplication principle of counting. <br> Main Dish <br> Cheeseburger Burrito Pizza <br> Drink <br> Milk <br> Water <br> Juice |  |


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|  |  |  |  | Both of the representations above illustrate a 3 - 3 relationship, which connects to the multiplication principle. Students explain where the multiplication principle appears in each representation. In this example, there are $3 \cdot 3=9$ possible meals. |  |
| 4.0A | B | 4 | Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range $1-100$ is a multiple of a given one-digit number. Determine whether a given whole number in the range $1-100$ is prime or composite. <br> 4.MP.2. Reason abstractly and quantitatively. <br> 4.MP.7. Look for and make use of structure. | Students should understand the process of finding factor pairs so they can do this for any number 1-100. <br> Example: <br> - Factor pairs for 96: 1 and 96, 2 and 48,3 and 32,4 and 24,6 and 16,8 and 12. <br> Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., $5,10,15,20$ (there are 4 fives in 20 ). <br> Example: <br> - Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 <br> Multiples $: 1,2,3,4,5 \ldots \underline{24}$ <br> $2,4,6,8,10,12,14,16,18,20,22, \underline{24}$ <br> $3,6,9,12,15,18,21, \underline{4}$ <br> $4,8,12,16,20, \underline{24}$ <br> 8,16,24 <br> 12,24 <br> $\underline{24}$ <br> To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following: <br> - all even numbers are multiples of 2 | Engage NY <br> M3 Lessons 22-25 <br> enVision <br> Topic 1,11 |


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|  |  |  |  | - all even numbers that can be halved twice (with a whole number result) are multiples of 4 <br> - all numbers ending in 0 or 5 are multiples of 5 <br> Prime vs. Composite: <br> - A prime number is a number greater than 1 that has only 2 factors, 1 and itself. <br> - Composite numbers have more than 2 factors. <br> Students investigate whether numbers are prime or composite by: <br> 0 building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, $1 \times 7$ and $7 \times 1$, therefore it is a prime number) <br> 0 finding factors of the number. |  |
| 4.NBT | B | 5 | Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. <br> 4.MP.2. Reason abstractly and quantitatively. <br> 4.MP.3. Construct viable arguments and critique the reasoning of others. <br> 4.MP.4. Model with mathematics. <br> 4.MP.5. Use appropriate tools strategically. <br> 4.MP.7. Look for and make use of structure. | Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the $5^{\text {th }}$ grade. <br> Students may use digital tools to express their ideas. <br> Use of place value and the distributive property are applied in the scaffold examples below. <br> - To illustrate $154 \times 6$ students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, $154 \times 6=(100+50+4) \times 6$ $=(100 \times 6)+(50 \times 6)+(4 \times 6)=600+300+24=924$. | Engage NY <br> M3 Lessons 34-38 <br> enVision <br> Topic 5,6,7,8 |


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|  |  |  |  | - The area model shows the partial products. $14 \times 16=224$ <br> Using the area model, students first verbalize their understanding: <br> - $10 \times 10$ is 100 <br> - $4 \times 10$ is 40 <br> - $10 \times 6$ is 60 , and <br> - $4 \times 6$ is 24 . <br> They use different strategies to record this type of thinking. <br> Students explain this strategy and the one below with base 10 blocks, drawings, or numbers. <br> - Matrix model <br> This model should be introduced after students have facility with the strategies shown above. |  |
| 4.NBT | B | 6 | Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies | In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context. | Engage NY <br> M3 Lessons 14-21, 26- <br> 33 |


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|  |  |  | based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. <br> 4.MP.2. Reason abstractly and quantitatively. <br> 4.MP.3. Construct viable arguments and critique the reasoning of others. <br> 4.MP.4. Model with mathematics. <br> 4.MP.5. Use appropriate tools strategically. <br> 4.MP.7. Look for and make use of structure. | Examples: <br> - A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box? <br> Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50 . <br> Using Place Value: $260 \div 4=(200 \div 4)+(60 \div 4)$ <br> Using Multiplication: $4 \times 50=200,4 \times 10=40,4 \times 5=20 ; 50+10+5=$ 65 ; so $260 \div 4=65$ <br> Students may use digital tools to express ideas. <br> - Using an Open Array or Area Model <br> After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the $5^{\text {th }}$ grade. <br> o Example 1: $150 \div 6$ <br> Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150 . <br> 1. Students think, 6 times what number is a number close to 150 ? They recognize that $6 \times 10$ is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60 . They express that they have only used 60 of the 150 so | enVision <br> Topic 9,10 |


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|  |  |  |  | they have 90 left. <br> 2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left. <br> 3. Knowing that $6 \times 5$ is 30 . They write 30 in the bottom area of the rectangle and record 5 as a factor. <br> 4. Students express their calculations in various ways: <br> a. 150 $150 \div 6=10+10+5=25$ $\begin{aligned} & \frac{-60}{90}(6 \times 10) \\ & \frac{-60}{30}(6 \times 10) \\ & -\quad 30(6 \times 5) \\ & \hline 0 \end{aligned}$ <br> b. $150 \div 6=(60 \div 6)+(60 \div 6)+(30 \div 6)=10+10+5=25$ <br> - Example 2: $1917 \div 9$ <br> A student's description of his or her thinking may be: <br> I need to find out how many 9s are in 1917. I know that $200 \times 9$ is 1800 . So if I use 1800 of the 1917 , I have 117 left. I know that $9 \times 10$ is 90 . So if I have 10 more 9 s , I will have 27 left. I can make 3 more 9 s . I have 200 nines, 10 nines and 3 nines. So I made 213 nines. $1917 \div 9=213$. |  |


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| In this 45-day unit, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations. |  |  |  |  |  |
| Major Clusters: |  |  | 4.NF.A - Extend understanding of fraction equivalence and ordering. <br> 4.NF.B - Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. |  |  |
| Supporting Clusters: |  |  |  |  |  |
| Vocabulary |  |  | Benchmark, common denominator, denominator, line plot, mixed number, numerator |  |  |
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| 4.NF | A | 1 | Explain why a fraction $a / b$ is equivalent to a fraction $(\mathrm{n} \times \mathrm{a}) /(\mathrm{n} \times \mathrm{b})$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. <br> 4.MP.2. Reason abstractly and quantitatively. <br> 4.MP.4. Model with mathematics. <br> 4.MP.7. Look for and make use of structure. <br> 4.MP.8. Look for and express regularity in repeated reasoning. | This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100). <br> Students can use visual models or applets to generate equivalent fractions. <br> All the models show $1 / 2$. The second model shows $2 / 4$ but also shows that $1 / 2$ and $2 / 4$ are equivalent fractions because their areas are equivalent. When a horizontal line is drawn through the center of the model, the number of equal parts doubles and size of the parts is halved. Students will begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions. $1 / 2 \times 2 / 2=2 / 4$ | Engage NY <br> M5 Lessons 7-11, 16-28 <br> enVision <br> Topic 11 |


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| 4.NF | A | 2 | Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. <br> 4.MP.2. Reason abstractly and quantitatively. <br> 4.MP.4. Model with mathematics. <br> 4.MP.5. Use appropriate tools strategically. <br> 4.MP.7. Look for and make use of structure. | Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths. <br> Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, $>,=$. <br> - Fractions may be compared using $\frac{1}{2}$ as a benchmark. <br> Possible student thinking by using benchmarks: <br> o $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. | Engage NY <br> M5 Lessons 12-15, 22- <br> 28 <br> enVision <br> Topic 11 |


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|  |  |  |  | o $\stackrel{5}{6}>\frac{1}{2}$ because $\underset{6}{\frac{3}{6}=\frac{1}{2}}$ and $\underset{6}{\underset{6}{6}} \frac{3}{6}$ <br> Fractions with common denominators may be compared using the numerators as a guide. <br> - $\frac{2}{6}<\frac{3}{6}<\frac{5}{6}$ <br> Fractions with common numerators may be compared and ordered using the denominators as a guide. <br> - $\frac{3}{10}<\frac{3}{8}<\frac{3}{4}$ |  |
| 4.NF | B | 3 <br> abc <br> d | Understand a fraction $a / b$ with $a>1$ as $a$ sum of fractions $1 / b$. <br> a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. <br> b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. <br> Examples: $3 / 8=1 / 8+1 / 8+1 / 8$; $3 / 8=1 / 8+2 / 8 ; 21 / 8=1+1+1 / 8=8 / 8+8 / 8$ $+1 / 8$. <br> c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. <br> d. Solve word problems involving addition and subtraction of fractions referring to | A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as $2 / 3$, they should be able to decompose the non-unit fraction into a combination of several unit fractions. <br> Examples: <br> Fraction Example 1: <br> - $2 / 3=1 / 3+1 / 3$ <br> Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding. <br> Fraction Example 2: <br> - $11 / 4-3 / 4=\square$ <br> $4 / 4+1 / 4=5 / 4$ <br> $5 / 4-3 / 4=2 / 4$ or $1 / 2$ <br> Word Problem Example 1: <br> Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate $2 / 6$ of the pizza. How much of the pizza did the girls eat together? <br> Solution: The amount of pizza Mary ate can be thought of a 3/6 or $1 / 6$ and $1 / 6$ and $1 / 6$. The amount of pizza Lacey ate can be thought of a $1 / 6$ and $1 / 6$. The total amount of pizza they ate is | Engage NY <br> M5 Lessons 1-11, 16-28 <br> enVision <br> Topic 12 |



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| 4.NF | B | $\begin{gathered} 4 \\ \mathrm{abc} \end{gathered}$ | Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. <br> a. Understand a fraction $a / b$ as a multiple of $1 / b$. For example, use a visual fraction model to represent 5/4 as the product $5 \times(1 / 4)$, recording the conclusion by the equation $5 / 4=5 \times(1 / 4)$. <br> b. Understand a multiple of $a / b$ as a multiple of $1 / b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times(2 / 5)$ as $6 \times(1 / 5)$, recognizing this product as 6/5. (In general, $n \times(a / b)=(n \times a) / b$.) <br> c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3 / 8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? <br> 4.MP.1. Make sense of problems and persevere in solving them. <br> 4.MP.2. Reason abstractly and quantitatively. | Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns. <br> Examples: <br> - $3 \times(2 / 5)=6 \times(1 / 5)=6 / 5$ <br> - If each person at a party eats $3 / 8$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie? <br> A student may build a fraction model to represent this problem. |  |  |  |  |  |  | Engage NY <br> M5 Lessons 1-6, 22-28, 35-40 <br> enVision <br> Topic 13 |


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|  |  |  | 4.MP.4. Model with mathematics. <br> 4.MP.5. Use appropriate tools strategically. <br> 4.MP.6. Attend to precision. <br> 4.MP.7. Look for and make use of structure. <br> 4.MP.8. Look for and express regularity in repeated reasoning. |  |  |

